Introducing the intermediate solution vector Z defined by

$$\begin{bmatrix} Z_k \\ Z_v \end{bmatrix} = \begin{bmatrix} U_{kk} & U_{kv}E \\ 0 & U_{vv}E \end{bmatrix} \begin{bmatrix} X_k \\ X_v \end{bmatrix} \tag{5}$$

Eq. (4) is rewritten as

$$\begin{bmatrix} L_{kk} & 0 \\ L_{vk} & L_{vv} \end{bmatrix} \begin{bmatrix} Z_k \\ Z_v \end{bmatrix} + \begin{bmatrix} 0 \\ DX_v \end{bmatrix} = \begin{bmatrix} B_k \\ B_v \end{bmatrix} \tag{6}$$

The efficient repetitive solution of Eq. (1) is based on three relations inherent in Eqs. (5) and (6). Substituting the second matrix equation of Eq. (5) into the second matrix equation of Eq. (6) and rewriting the remaining equations yields

$$L_{kk}Z_k = B_k \tag{7}$$

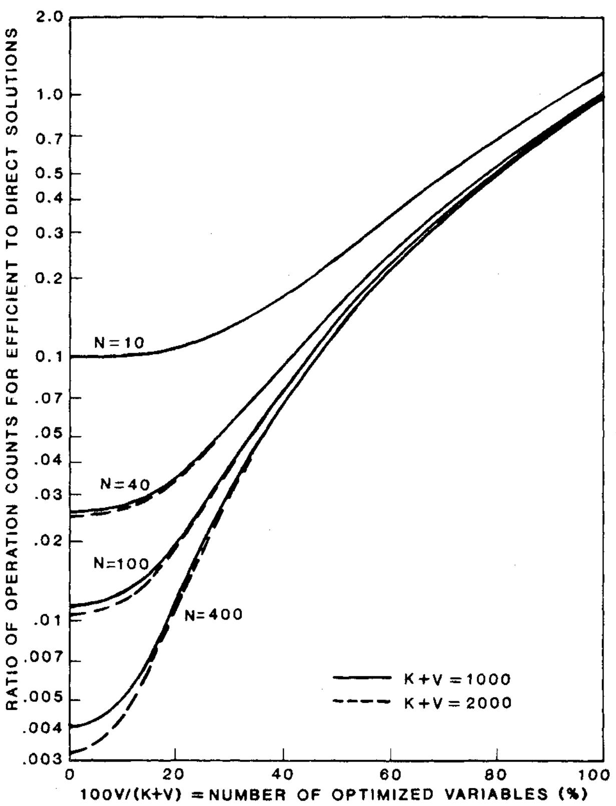

Fig. 1 Ratio of operation count estimates for efficient to direct solutions.

Table 1 Direct solution operation count

Operation	Operation count
Factor M	$N(K+V)^3/3$
Backsolve for X	$N(K+V)^2$

Table 2 Efficient repetitive solution operation count

Operation	Operation count
Factor A	$(K+V)^3/3$
Backsolve Z, k set	$K^2/2$
Form $B-LZ$	$KV+V$
Multiply LU	$V^3/3$
Multiply $(LU)E+D$	NV^2
Solve for X, v set	$N(V^3/3+V^2)$
Solve for X, k set	$N(K^2/2+KV+K+V)$

$$[L_{vv}U_{vv}E+D]X_v = B_v - L_{vk}Z_k \tag{8}$$

$$U_{kk}X_k = Z_k - U_{kv}EX_v \tag{9}$$

Equation (7) is solved for Z_k, Eq. (8) for X_v, and Eq. (9) for X_k. The solution for Z_k, the right-hand side of Eq. (8), and the LU product of Eq. (8), are required only once since they do not vary with E or D. The solution of Eqs. (8) and (9) must be obtained repetitively. The savings in computational effort result from factoring and backsolving Eq. (8), as opposed to Eq. (1).

Operation count estimates for the direct solution of a set of linear equations and the efficient repetitive solution presented herein are given in Tables 1 and 2. The count estimates are developed as a function of K, the number of degrees of freedom in the k set; V, the number of degrees of freedom in the v set; and N, the number of solutions to the system.

The ratio of operation counts for the efficient repetitive solution vs the direct solution of linear equations is presented in Fig. 1. These data were generated using the estimates of Tables 1 and 2 for a system of equations of order $K+V=1000$ and 2000. Data are presented for the $N=10$, 40, 100, and 400 solutions. Results for a system of equations of order 2000 are quite similar to the results of order 1000. Data for $K+V=3000$ have been evaluated and exhibit similar characteristics. Operation count ratios in the preceding problem size ranges are, practically speaking, independent of the size of the linear system.

The results of Fig. 1 may be interpreted as follows for the problem sizes considered. If the number of variable degrees of freedom in a linear system (v set) is less than 10% of the total, the cost (in operation count) of all additional repetitive solutions is less than the cost of the original solution.

The Role of Damping on Supersonic Panel Flutter

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Introduction

ONE of the most important and interesting aspects of the theory of stability of elastic systems subjected to nonconservative forces is connected with the damping effects. In dealing with a double mathematical pendulum subjected on a free end to a follower (tangential) force, Ziegler[1] has found that the addition of a small amount of damping can reduce the value of the critical force in comparison with the value found without taking the damping into account. The effects of damping on the stability of elastic systems subjected to nonconservative forces have been studied by many authors. Some general results related to the stability of nonconservative systems have been obtained by Nemat-Nasser et al.[2,3] Bolotin and Zhinzher[4] conducted a systematic study of the damping effects on the stability of finite-degree-of-freedom and continuous systems subjected to nonconservative forces. A principal conclusion of this paper was that "for real laws of damping a considerable part of quasistability region belongs to the instability region. From this rigorous point of view the major-

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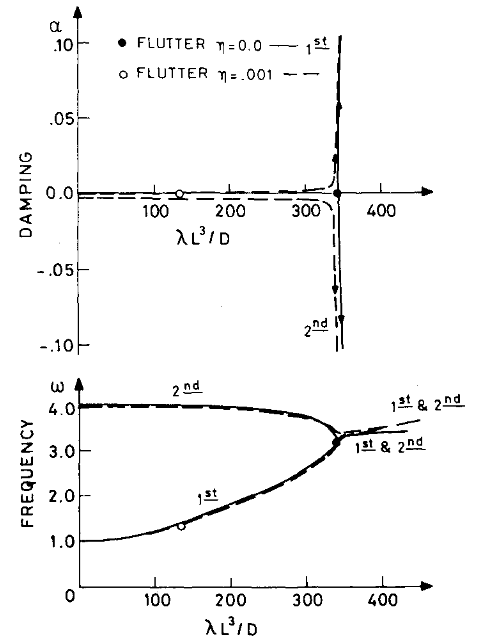

Fig. 1 Variation of the damping and frequencies of the first two modes of the plate for increasing dynamic pressure of the flow for $\eta=0$ and $\eta=0.001$.

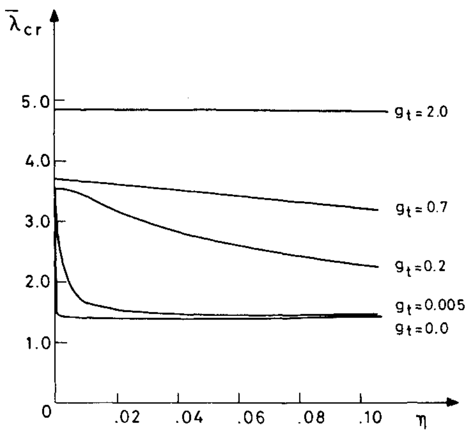

Fig. 2 Variation of the critical dynamic pressure for increasing viscoelastic damping for several values of external damping coefficients.

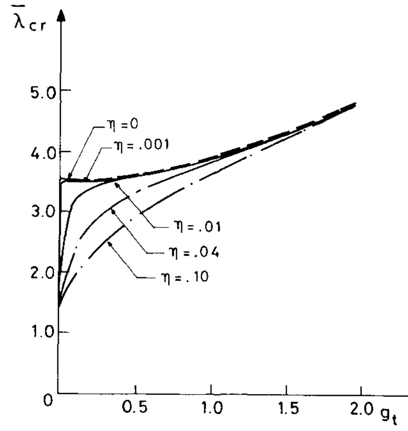

Fig. 3 Variation of the critical dynamic pressure for increasing external damping for several values of viscoelastic damping coefficients.

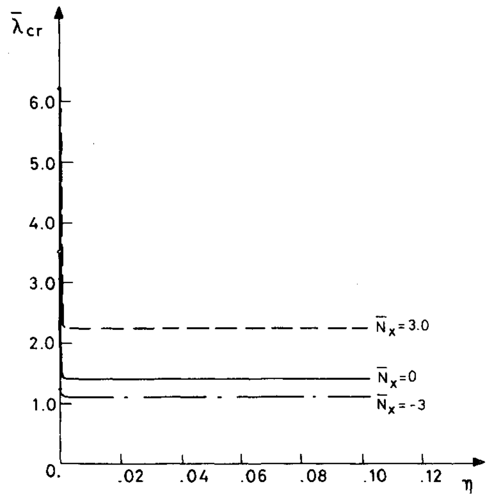

Fig. 4 Variation of the critical dynamic pressure vs viscoelastic damping for several values of midplane forces.

ity of papers dealing with stability of nonconservative systems (including panel flutter) need to be reconsidered."

The present Note will deal with the effect that viscoelastic damping has on the instability of the plate at supersonic flow. Some aspects of the viscoelastic damping coupled with midplane forces or external damping are considered.

Basic Panel Flutter Equation and Its Solution

Consider a flat, simply supported, one-dimensional plate (beam) subject to a supersonic flow over one side. The nondimensional governing differential equation for the panel flutter problem is[5]

$$\left(1+\eta\frac{\partial}{\partial\tau}\right)\frac{\partial^4 w}{\partial\xi^4}-\frac{N_xL^2}{D}\frac{\partial^2 w}{\partial\xi^2}+\frac{\lambda L^3}{D}\frac{\partial w}{\partial\xi}$$
$$+\pi^4 g_t\frac{\partial w}{\partial\tau}+\pi^4\frac{\partial^2 w}{\partial\tau^2}=0 \tag{1}$$

where η is the coefficent of internal dissipation (assumed to be of a viscoelastic type) and g_t the sum of the aerodynamic damping and the effective structural damping coefficient.

Seeking solutions of the form

$$w(\xi,\tau)=e^{(\alpha+i\omega)\tau}\sum_{k=1}^{4}C_ke^{r_k\xi} \tag{2}$$

where α and ω are the damping and frequency (assumed real) of the mode shape and r_k the roots of the characteristic equation

$$[1+\eta(\alpha+i\omega)]r^4-\frac{N_xL^2}{D}r^2+\frac{\lambda L^3}{D}r$$
$$+\pi^4 g_t(\alpha+i\omega)+\pi^4(\alpha+i\omega)^2=0 \tag{3}$$

The constants C_k should be adjusted so as to satisfy the simply supported edges conditions, yielding

$$\sum_{k=1}^{4}C_k=0,\quad \sum_{k=1}^{4}r_k^2C_k=0$$
$$\sum_{k=1}^{4}e^{r_k}C_k=0,\quad \sum_{k=1}^{4}r_k^2e^{r_k}C_k=0 \tag{4}$$

To obtain a nontrivial solution, the determinant Δ of the system of the linear equation (4) must equal zero.

For the panel to undergo flutter instability, one has to seek the combination of ω and λ (minimum) that will fulfill Eq. (3) and $\Delta=0$, rendering the system from neutral stability ($\alpha=0$) to instability ($\alpha>0$). The detailed procedure for solving the problem is stated in Ref. 6.

Applications

Assuming $\eta=N_x=\lambda=g_t=0$, the panel flutter problem is reduced to a simply supported beam and thus the natural frequencies are well known. Slowly increasing the dynamic pressure λ of the flow ($\eta=N_x=g_t=0$), one can trace the behavior of, say, the first two frequencies, as shown in Fig. 1. The first two modes approach each other ($\alpha=0$) and, at $\lambda_{cr}L^3/D=343.36$ (known value, see Ref. 7), coalesce to render one mode unstable ($\alpha>0$). The behavior of the natural frequencies vs $\lambda L^3/D$ for a very light damped panel ($\eta=0.001$) are plotted in Fig. 1 (dashed lines). For low dynamic pressure, the modes are damped ($\alpha<0$) and at $\lambda_{cr}L^3/D=134.93$ the first mode becomes unstable ($\alpha>0$). The most striking result deduced from Fig. 1 is the sharp reduction in λ_{cr} for the lightly damped case $\eta=0.001$ compared to the undamped case $\eta=0$ (see Ref. 4). A confirmation of this unexpected reduction in λ_{cr} for the very lightly damped panel case was also obtained by applying a three-mode Galerkin approximation.

These interesting theoretical results were followed by an investigation of the coupling effect between the viscoelastic η and external damping g_t coefficients on the panel flutter instability. Figure 2 represents the critical dynamic pressure $\bar{\lambda}_{cr}(=\lambda L^3/D\pi^4)$ vs η for several values of g_t. The results show the destabilizing effect of the viscoelastic damping η for several values of g_t. Figure 3 displays the variation of $\bar{\lambda}_{cr}$ vs g_t for several values of viscoelastic damping η. It is seen that g_t has a stabilizing effect for all values of η. The results of Figs. 2 and 3 emphasize the strong interaction between the viscoelastic and external (viscous) damping on the panel flutter instability. It should be noted that the three-term Galerkin approximation confirms this finding.

Another course of investigation was to determine if there is a coupling between the damping and the midplane forces exerted on the plate. Figure 4 shows results obtained for $\bar{\lambda}_{cr}$ vs η for a plate subjected to a tension or compression midplane force of $\bar{N}_x(=N_xL^2/D\pi^2)=\pm3$. Again, it can be seen that viscoelastic damping has a strong destabilizing effect for all values of $\eta\neq0$.

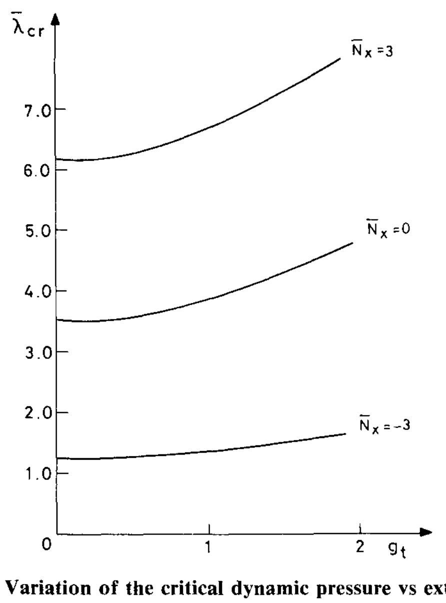

Fig. 5 Variation of the critical dynamic pressure vs external damping for several values of midplane forces.

Figure 5 plots $\bar{\lambda}_{cr}$ vs g_t for several values of the midplane forces while $\eta=0$. g_t has a stabilizing effect on the panel flutter. (The results for $\bar{N}_x=0$ are identical to those obtained in Ref. 8.)

References

[1]Ziegler, H., "Die Stabilitätskriterien der Elastomechanik," *Ingenieur-Archiv*, Vol. 20, No. 1, 1952, pp. 49-56.

[2]Nemat-Nasser, S., Prasad, S. N., and Herrmann, G., "Destabilizing Effect of Velocity-Dependent Forces in Nonconservative Continuous System," *AIAA Journal*, Vol. 4, No. 7, 1966, pp. 1276-1280.

[3]Nemat-Nasser, S. and Herrmann, G., "Some General Considerations Concerning the Destabilizing Effect in Nonconservative Systems," *Zeitschrift fuer Angewandte Mathematik und Physik*, Vol. 17, 1966, pp. 305-313.

[4]Bolotin, V. V. and Zhinzher, N. I., "Effects of Damping on Stability of Elastic Systems Subjected to Nonconservative Forces," *International Journal of Solids and Structures*, Vol. 5, 1969, pp. 965-989.

[5]Bisplinghoff, R. L. and Ashley, H., *Principles of Aeroelasticity*, John Wiley & Sons, New York, 1962, Chap. 8.

[6]Lottati, I. and Kornecki, A., "The Effect of an Elastic Foundation and of Dissipative Forces on the Stability of Fluid-Conveying Pipes," Technion, Haifa, Israel, Rept. TAE No. 563, Feb. 1985.

[7]Hedgepeth, J. M., "Flutter of Rectangular Simply Supported Panels at High Supersonic Speeds," *Journal of the Aeronautical Sciences*, Vol. 24, No. 8, 1957, pp. 563-573.

[8]Houbolt, J. C., "A Study of Several Aerothermoelastic Problems of Aircraft Structures," *Mitteilungen aus dem Institut fur Flugzeugstatik und Leichtbau*, No. 5, E.T.H., Zürich, 1958.

Optimization of Superplastically Formed Sandwich Cores

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Introduction

SUPERPLASTIC forming with or without diffusion bonding has proved to be a promising and innovative approach to manufacturing components with complicated shapes in many challenging engineering problems. This new forming process can improve material utilization, simplify machining and assembly, and reduce production costs. It is accomplished through a "one-piece" forming concept that enables the number of parts and fasteners to be greatly reduced. This new technology has been used successfully in different branches of structural engineering, including sophisticated aircraft design, turbine gear manufacture and ground transportation vehicle design. This is confirmed by a recent summary article[1] reporting that more than 200 components in nine current U. S. aircraft and spacecraft (including the B-1B, F-15, F-18A, and Space Shuttle) have been formed superplastically.

Superplastic forming uses a special mechanical property of certain metallic materials that exhibit exceptionally high duc-

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